

SyMan 1.2

a SYmbolic MANipulator

by Roy Wood

Introduction:

Most mathematics programs you have ever worked with have likely been little more than fancy calculators with maybe some useful extras, like a graphing package attached. These programs are great for playing with numbers and graphing specific equations, but that's about it. SyMan, on the other hand, does all of the things these standard programs do, but it goes one step further: SyMan knows algebra. (sorry, Bo!)

SyMan is what academic-types call a "symbolic manipulator." This means that SyMan is basically an algebraic word-processor. You can enter just about any algebraic expression, and then manipulate it in just about any way you please. For example, you can enter the equation " $6x + 7 = -x + 14$ " and eventually work it into the form " $x = 1$ ". SyMan takes care of moving all the symbols around correctly, and all you have to do is tell it to do things like "add x," "subtract 7," and "divide 7."

Just as a word processor allows you to freely rearrange your words and experiment with different ways of expressing your ideas, so too does SyMan allow you to easily manipulate and explore mathematical expressions and concepts. SyMan does not absolve you of the need for mathematical understanding and intelligence; it simply takes care of the drudgery of moving numbers and variables around on the page. With SyMan you can finally concentrate on the mathematics of a problem, rather than the algebra and arithmetic. And isn't that why it's called "mathematics?"

A Basic Example:

SyMan is a standard Macintosh program, so you should be familiar with using the Macintosh (mouse, menus, etc.). If you can figure out what the following phrase means, you are probably expertly-qualified to use SyMan: double-click the SyMan icon to launch the program.

The best way to learn the basics of using SyMan is through a hands-on example, so try following along with this one:

- 1.) Using SyMan consists of entering an expression, and then using the menu commands to manipulate it. If you are menu-shy or mouse-shy, you can also type the commands (if you are a good typist, this is usually faster anyway). Type " $6x + 7 = -x + 14$ " (no quotation marks around the equation, of course).
- 2.) As you probably noticed, all typing shows up in the text window at the lower left half of the screen. And if you pressed return or enter (try it if you didn't), you probably also noticed that when you enter an expression, it is displayed in LARGE PRINT in the upper right corner of the screen.

Now that we've entered our equation, let's solve for the value of 'x'.

- 3.) First, pull down the "Manipulate" menu and choose the "Add" option. You'll be asked what you want to add, and since we want to get rid of the '-x' on the right side of the equation, tell the computer to add 'x'. (You could also skip the menu selection, and just type "add x" followed by return.) If all goes well, the active equation (the LARGE PRINT one in the top right corner) should be " $6x + 7 + x = -x + 14 + x$ " after 'x' is added to both sides.

- 4.) Obviously the active equation can be simplified, so pull down the "Simplify" menu and choose the "Simplify" option (or just type "simp" and press return). The active equation should become " $7x + 7 = 14$ ".
- 5.) Next, we want to get rid of the '7' from the left side of the equation. We do this the same way we got rid of the '-x' from the right side: by either adding '-7' or subtracting '7' from both sides of the equation. You can use the "Add" or "Subtract" options under the "Manipulate" menu, or you can type the "add" or "subt" commands. You should also simplify the result.
- 6.) Finally, the active equation should be " $7x = 7$ ", and all that's left to do is divide both sides by '7'. Use the "Divide" option under the "Manipulate" menu, or type the "div" command. Alternatively, you can try using the small buttons in the top left corner; they are much easier than either menus or typed commands.

If everything worked, the final active equation should be " $x = 1$ ", and you should have learned how to enter an equation and manipulate it. These few skills are the essence of everything you do with SyMan.

Now, If you're like me, even a simple example like this is overwhelming when you have to contend with an unfamiliar and possibly intimidating computer program. To help you gain confidence, pull out a grade 9 or 10 math textbook and practice using SyMan to solve some of the algebra problems listed. And if you're feeling cocky, try some higher-order equations.

As for the rest of this manual, it just describes the technical features of SyMan, and how to make the most of the program (including some shortcuts and useful tips!). The best way to learn to use SyMan is by using it, so experiment with the program first, and then glance at the manual later.

SyMan Reference Guide:

General Information:

SyMan's powerful parser understands numbers, π , the variables 'a' through 'z', the usual +, -, /, * operators, the caret '^' exponentiation operator, the '\sqrt' root symbol (option-v), brackets, implied multiplication, unary minus, and functions like `SIN()`, `COS()`, etc. (recognized functions are listed at the end of this manual).

Numbers are stored internally as rationals for numbers in reasonable ranges, so round-off errors don't normally occur (not true for very large or very small magnitudes). SyMan knows fractions!

Specific parts of an expression may be manipulated by selecting them before issuing the command. Select a sub-expression by clicking or click-dragging on it in the active equation window in the top-right corner of the screen; the selected sub-expression will be highlighted in black. You will find it very useful to be able to operate on a sub-expression, rather than the entire active equation!

Manipulate commands prompt for the expression to add, subtract, etc. The expression may be indicated by selecting it in the active equation window before issuing the manipulate command.

The selected expression will appear as the default input in the dialog box, and all the user need do is press return or click on the "OK" button to accept. And if you hold down the option key as you make the menu selection or press the shortcut button, SyMan will not even bother you with the dialog box.

Typed commands may be used in place of the mouse/menu. A list of the commands is provided at the end of this document.

The remainder of this reference guide is subdivided into topics relating to the functional parts of the screen— i.e. the text window, the graph window, the active equation, and the menus.

The Text Window

The text window (located in the lower left half of the screen) serves a twofold purpose: it allows you to enter expressions and commands, and it records the work you have done. The text window may be scrolled through using the scroll bars or arrow keys, and you may cut and paste freely using the "Edit" menu" or standard command keys. As well, you may save, load, or print the contents of the text window using the commands under the "File" menu. Note that the current window must be closed before another one may be opened (use "Close" under the "File" menu).

Commands or expressions are entered by pressing the return key, and old commands or expressions may be re-entered by moving the cursor to an old line and pressing return. This feature is particularly useful for backing up to a previous expression— simply cut out the text back up to the desired expression, then press return on that line.

To simply insert a blank line in the text window, press shift and return simultaneously— this tells SyMan to ignore the text of the current line. Also, comments may be entered by placing a bullet character ("•") at the start of a line.

A list of recognized typed commands (as opposed to menu commands) may be obtained by giving the command "help". Generally, the commands are similar in name to the related menu commands.

The Graph Window:

The graph window is 256 pixels wide and 256 pixels tall. Cursor coordinates are displayed in the top right corner of the graph window.

SyMan can graph any function you can enter. The "Graph" menu contains all the options related to graphing; see that heading under the Menu section of this reference guide. Generally, equations should be in $y=f(x)$ form, or else be a standard-form conic equation like $y = \sqrt{f((x-2)\sqrt{s(2),4}) + \sqrt{f((y+1)\sqrt{s(2),9})} = 1$. Implicitly-defined equations are also acceptable, though they take much longer to graph.

You may zoom-in on the graph window by click-dragging an outline of the desired zoom region inside the graph window. As well, the limits may be set via the "Graph Options" option under the "Graph" menu.

The vertical and horizontal axes variables are displayed, respectively, in the upper-left and lower-right corners of the graph window. These may be changed via the "Graph Options" option under the "Graph" menu. As well, the horizontal axis may be displayed in units of π .

The drawing of a graph may be interrupted by pressing command-period.

Active Equation:

The active equation is displayed in the upper right corner of the screen.

The active equation is that expression upon which all manipulation commands operate, and that which the "Graph Active Eqn" command will graph.

Specific parts of the active equation may be selected by clicking or click-dragging on them with the arrow cursor. Selected sub-expressions are highlighted in black. Simplify commands operate on the selected sub-expression, or if no sub-expression is selected, the entire equation is operated upon. Selecting a sub-expression allows you greater control over manipulating the expression (for example, a single term can be factored, rather than factoring every term in the entire equation). As well, if a sub-expression is selected, it is used as the default input to the manipulate commands.

Shortcut Buttons:

The shortcut buttons are located in the upper left corner of the screen.

The shortcut buttons are simply a fast way of giving the most frequently used SyMan menu commands. A brief description of each is listed below; for more information, see the corresponding menu description.

| | |
|----------|--|
| '+' | "Add" manipulate command |
| '-' | "Subtract" manipulate command |
| 'x' | "Multiply" manipulate command |
| '÷' | "Divide" manipulate command |
| '√' | apply square-root to active expression/equation |
| 'raise' | "Raise To" manipulate command |
| 'apply' | "Apply Function" manipulate command |
| 'simp' | "Simplify" simplify command |
| 'expand' | "Expand" simplify command |
| 'factor' | "Factor" simplify command |
| 'remem' | "Remember Active Eqn" command from Variable menu |
| 'subst' | "Substitute" for 'remembered' variable |

Other Buttons The last four "Remembered" equations are accessible for quick substitution via these buttons.

Use the option key to take real advantage of the shortcut buttons. For example, if the active equation is " $3x + 7y = 11$ ", you can subtract " $7y$ " by clicking on the " $7y$ " in the active equation window and then pressing the "-" button while holding down the option key. This avoids having to deal with the dialog box that asks you what you want to subtract.

Menus

Menu

About SyMan— displays a nifty information box giving my name, where to send your money, etc.

File Menu

| | |
|-------------|--|
| New— | opens up a new, empty workscreen (Close old one first) |
| Open— | opens up an old, saved workscreen (Close old one first) |
| Close— | closes current workscreen |
| Save— | save current workscreen under current name |
| Save As— | saves current workscreen under a new name |
| Revert— | discards changes to current workscreen, reverts to most recently-saved version |
| Page Setup— | sets up printer settings |
| Print— | print out contents of current text window (not graph) |
| Quit— | quit SyMan |

Edit Menu

| | |
|----------------|---|
| Undo— | sorry! not yet implemented! |
| Cut— | cut selected text-window text to clipboard |
| Copy— | copy selected text-window text to clipboard |
| Paste— | paste to text window from clipboard |
| Clear— | clear selected text |
| Select All— | select all text-window text |
| Copy Graph— | copy the graph to the clipboard |
| Edit Equation— | an awkward editor that I gave up on :-) |

Manipulate Menu

Note: *Manipulate commands prompt for the expression to add, subtract, etc. This expression may be indicated by selecting it in the active equation window before issuing the manipulate command. The selected expression will appear as the default input in the dialog box, and all the user need do is press return or click on the "OK" button to accept. And for the real power-user, hold down the option key as you make the menu selection or click the shortcut button; the dialog box will not even be displayed.*

| | |
|-----------|-------------------------------|
| Add— | add to active equation |
| Subtract— | subtract from active equation |
| Multiply— | multiply active equation |
| Divide— | divide active equation |

| | |
|--------------|---|
| Raise To— | raise active equation to some exponent |
| Apply— | apply function to active equation |
| Add Zero— | add an expression that simplifies to zero— an action which is actually useful for doing such odd things as completing squares, etc. |
| Mult By One— | multiply by an expression that simplifies to one— another strange action that proves useful for rationalizing denominators, etc. |

Simplify Menu

| | |
|---------------|--|
| Simplify— | collect, reduce the active equation or selected sub-expression of active equation |
| Collect— | collect like terms in the active equation or selected sub-expression of active equation |
| Expand— | expand all terms in the active equation or selected sub-expression of active equation |
| Reduce— | reduce all terms in active equation or selected sub-expression of active equation |
| Factor— | factor a given expression out of the active equation or selected sub-expression of active equation |
| Switch Order— | switch the left/right sides of the active equation or left/right sides of the selected sub-expression |
| Get Left— | discard right side of an equation, keep left side |
| Get Right— | discard left side of an equation, keep right side |
| Equate To— | set the active expression equal to another user-specified expression |
| Eval— | show value when active equation or selected sub-expression is evaluated |
| Select More— | select more of the sub-expression of the active equation (use the mouse for better control when selecting in the active equation window) |

Variables Menu

| | |
|---------------|--|
| Substitute— | substitute some expression for a variable (click on the variable to substitute for, then hold down the option key to dispense with the dialog box if a "Remember Eqn" operation has previously been performed) |
| Values— | show or set the current value of a variable |
| Remember Eqn— | store the active equation in memory so that it may be substituted later (useful for solving a system of equations). Up to four variables may be "remembered" this way. |

Remem Eqns— examine/change the substitutions noted via the "Remember Eqn" option, the "remem" button, or the "remem" command

Graph Menu

Graph Active— graph the current active equation and add that equation to the list of the four most-recently-graphed equations (equation must be in $y=f(x)$ form; standard-conic form $\frac{(x-2)^2}{4} + \frac{(y+1)^2}{9} = 1$; or implicit form)

Graph All— redraw the four most-recently-graphed equations

Equations— examine or change the record of the four most-recently-graphed equations

Table Of Values— generate a table of values for the current active equation

Draw Point— draw a single point somewhere on the grid (hold down the option key to draw more than one point)

Draw Line— draw a straight line on the graph (hold down the option key to draw more than one line)

Draw Freehand— draw a freehand line on the graph (hold down the option key to draw more than one line)

Graph Options— set the limits on the graph (you can also drag an outline rectangle in the graph window to zoom in), set axes ordinate/abscissa labels, enable/disable gridlines on the graph, and enable/disable marking of the graph scale.

Zoom Out— zoom out by a factor of 2, preserving the center point of the graph

Reset Limits— reset the limits on the graph to their default values of ± 5 .

Clear Graph— clear the graph window

Iteration Menu

Note: *Each iteration menu choice gives you the option of connecting points, clearing the old display before beginning graphing, and click-stepping through the points. "Click-stepping" means the computer will display the current (x,y) iteration point and then wait until the user clicks the mouse button before it continues with the iteration.*

To disable click-stepping during an iteration, hold down the command key and click the mouse button. To enable click-stepping during an iteration, just click the mouse button.

Any iteration may be interrupted by typing command-period on the keyboard (much like a graph may be interrupted).

Two Variables— iterate a pair of variables according to user-specified transformations, graph the resulting points

Try $x = (\cos(y) - \pi) * \cos(x)$
 $y = (\cos(y) - \pi) * \sin(x)$

Four Variables— iterate four variables according to user-specified transformations, and graph the resulting (x,y) points

Try $x = x + u$
 $u = u - 0.1x / \sqrt{x^2 + y^2}^3$
 $y = y + v$
 $v = v - 0.1y / \sqrt{x^2 + y^2}^3$

For better accuracy, use

$$x = x + u - 0.1x / \sqrt{x^2 + y^2}^3$$

$$y = y + v - 0.1y / \sqrt{x^2 + y^2}^3$$

Play with the gravitational constant, "0.1," and the initial x/y/u/v values.

Orbit Diagram— draw an orbit diagram for a user-specified function

See "Chaos, Fractals, and Dynamics" by Devaney, or try

$$x = c$$

$$y = y * y + c$$

with $c = -2$ to 0.25 step 0.008789 . (Note that $0.008789 = 2.25/256$, which is the optimal step size for the graph width of 256 pixels.)

Julia Set— the classic oh-so-slow Julia Set algorithm
Typed Commands:

add add some expression to the active equation. e.g. "add $7x - 4$ "

subt subtract some expression from the active equation. e.g. "subt $8x - 3$ "

mult multiply active equation by some expression. e.g. "mult $2x$ "

div divide active equation by some expression. e.g. "div $x/2$ "

raiseto raise active equation to some expression. e.g. "raiseto $1/2$ "

apply apply function to active equation. e.g. "apply SIN"

simp simplify active equation

collect collect like terms in active equation

expand expand all products in active equation

reduce reduce all terms in active equation

factor factor an expression out of the active equation. e.g. "factor $x+1$ "

switch switch selected terms or entire active equation

| | |
|--------|---|
| getl | get left side of active equation |
| getr | get right side of active equation |
| val | show value of active equation when all variables substituted, functions applied |
| remem | store the active equation in memory so that it may be substituted later |
| subst | substitute some expression for a variable. e.g. "subst x=9+SIN(π)" Also, "subst" with only a variable specified (e.g. "subst x") implies that the expression stored by a "remem" command should be substituted. |
| ? | show the value of some variable. e.g. "? x" |
| let | set the value of some variable. e.g. "let x=9" |
| graph | graph the active equation |
| clear | clear the graph window |
| lims | set the graph limits. e.g. "lims -10,5,-1,8" |
| colour | set the "colour" for the next graph. e.g. "colour 3" |

Recognized Functions:

Note: all functions are expected to have their arguments contained in brackets. e.g. $\sin(x)$, $\ln(x+1)$, $\sqrt{x^2 + y^2}$, etc.

| | |
|----------|--|
| SIN(x) | returns the circular sine of 'x', 'x' in radians |
| COS(x) | returns the circular cosine of 'x', 'x' in radians |
| TAN(x) | returns the circular tangent of 'x', 'x' in radians |
| SINH(x) | returns the hyperbolic sine of 'x' |
| COSH(x) | returns the hyperbolic cosine of 'x' |
| TANH(x) | returns the hyperbolic tangent of 'x' |
| EXP(x) | returns $e^{s(x)}$, $e=2.71828182\dots$ (you can also use e^x as long as you don't change the value of the variable 'e' from its default value) |
| LN(x) | returns the natural logarithm of 'x' |
| LOG(x) | returns the base 10 logarithm of 'x' |
| SQRT(x) | returns the square-root of 'x' |
| FLOOR(x) | returns the next-smallest integer less than 'x' |
| CEIL(x) | returns the next-largest integer greater than 'x' |
| RAND(x) | returns a random number, r , such that $0 \leq r < x$ |

| | |
|-----------|--|
| NOT(x) | returns zero if 'x' is non-zero, returns 1.0 if 'x' is zero |
| IFPLUS(x) | returns 1.0 if 'x' is greater than zero, returns zero otherwise |
| IFNEG(x) | returns 1.0 if 'x' is less than zero, returns zero otherwise |
| IFZERO(x) | returns 1.0 if 'x' is zero, returns zero otherwise (like NOT(x)) |